

An improved model for the nonlinear velocity power spectrum

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ABSTRACT

The velocity divergence power spectrum is a key ingredient in modelling redshift space distortion effects on quasi-linear and nonlinear scales. We present an improved model for the $z = 0$ velocity divergence auto and cross power spectrum which was originally suggested by Jennings et al. 2011. Using numerical simulations we measure the velocity fields using a Delaunay tessellation and obtain an accurate prediction of the velocity divergence power spectrum on scales $k < 1h\text{Mpc}^{-1}$. We use this to update the model which is now accurate to 2% for both $P_{\theta\theta}$ and $P_{\theta\delta}$ at $z = 0$ on scales $k < 0.65h\text{Mpc}^{-1}$ and $k < 0.35h\text{Mpc}^{-1}$ respectively. We find that the formula for the redshift dependence of the velocity divergence power spectra proposed by Jennings et al. 2011 recovers the measured $z > 0$ $P(k)$ to markedly greater accuracy with the new model. The nonlinear $P_{\theta\theta}$ and $P_{\theta\delta}$ at $z = 1$ are recovered accurately to better than 2% on scales $k < 0.2h\text{Mpc}^{-1}$. Recently it was shown that the velocity field shows larger differences between modified gravity cosmologies and ΛCDM compared to the matter field. An accurate model for the velocity divergence power spectrum, such as the one presented here, is a valuable tool for analysing redshift space distortion effects in future galaxy surveys and for constraining deviations from general relativity.

Key words: Methods: N -body simulations - Cosmology: theory - large-scale structure of the Universe

1 INTRODUCTION

In the hierarchical model of structure formation, gravitational collapse and the accelerating cosmic expansion are two competing effects which determine the rate at which structures, such as galaxies and clusters, grow in the Universe. In addition to the Hubble flow, galaxies possess peculiar velocities, arising from inhomogeneities in the local density field, which can be used to probe the growth rate of structure (Percival *et al.* 2007; Guzzo *et al.* 2008; Blake *et al.* 2010, 2011; Schlegel *et al.* 2007; Beutler *et al.* 2012). These peculiar velocities distort the clustering signal along the line of sight giving rise to redshift space distortions (see e.g. Kaiser 1987; Hamilton 1998). It has been shown that a key ingredient to improve models of the power spectrum in redshift space is the inclusion of the nonlinear velocity divergence power spectrum (Scoccimarro 2004; Jennings, Baugh & Pascoli 2011a,b). In this paper we present an improvement to the model for both the auto and cross velocity divergence power spectrum presented in

Jennings, Baugh & Pascoli (2011a). The refinement of the model is driven by using a volume weighted Delaunay tessellation method to measure the nonlinear velocity fields to greater accuracy and to smaller scales than presented in Jennings, Baugh & Pascoli (2011a).

Measurements of the growth rate of structure can be used to determine if the accelerating expansion is the result of a dark energy component which behaves as a repulsive form of gravity or if Einstein’s theory of gravity breaks down on cosmological scales (see e.g. Bertschinger & Zukin 2008). Independent measurements of the growth rate can be obtained by measuring the clustering of galaxies in redshift space and recently there has been renewed interest in improving the models for the clustering signal in redshift space (Scoccimarro 2004; Percival & White 2009; Taruya *et al.* 2010; Jennings, Baugh & Pascoli 2011a; Seljak & McDonald 2011; Tang *et al.* 2011; Reid & White 2011; Kwan, Lewis & Linder 2012). Also recently it has been pointed out that the velocity divergence power spectrum is a more sensitive probe of modified gravity than the nonlinear matter power spectrum (Jennings *et al.* 2012) but models for the 2D redshift space power spectrum are currently not

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accurate enough to allow this quantity to be extracted to a high precision. However, given an accurate model for the velocity divergence power spectrum in a Λ CDM cosmology, this can be used to construct predictions for the moments of the power spectrum in redshift space which can be directly compared with data and has been shown to be accurate in detecting deviations from general relativity using simulations (Jennings, Baugh & Pascoli 2011b; Jennings *et al.* 2012).

The linear continuity equation, $\nabla \cdot v = -aHf\delta$, gives a one to one correspondence between the velocity and density fields where the linear growth rate $f = d\ln\delta/d\ln a$ is a cosmology dependent factor, δ is the matter overdensity, v is the peculiar velocity and H is the Hubble parameter. Once overdensities become nonlinear, this relationship no longer holds. Bernardeau (1992) derived the nonlinear relation between δ and $\nabla \cdot v$ in the case of an initially Gaussian field. Many authors since then have extended this relation using e.g. higher order perturbation theory or the spherical collapse model (see e.g. Scoccimarro & Frieman 1996; Chodorowski & Lokas 1997; Kudlicki *et al.* 2000; Bilicki & Chodorowski 2008; Kitaura *et al.* 2011). Cielieg & Chodorowski (2004) were the first to apply numerical simulations to model the nonlinear velocity and density spectra, instead of using perturbation theory. Using a grid-based pressureless hydrodynamic code their simulations had limited resolution but were able to fit the ratios up to $k = 1h\text{Mpc}^{-1}$ and show that perturbative predictions fail at $k > 0.2h\text{Mpc}^{-1}$.

The density velocity relation considered in this paper was first presented in Jennings, Baugh & Pascoli (2011a), where the parameters were obtained by fitting to a mass weighted measurement of the velocity divergence field. In this paper we present updated parameters obtained by fitting to the volume weighted velocity divergence power spectra which have been measured accurately on smaller scales than was possible with the mass weighted method (Pueblas & Scoccimarro 2009). We also show that the model proposed by Jennings, Baugh & Pascoli (2011a) for the redshift dependence of the density velocity relation recovers the velocity power spectra at $z > 0$ with greater precision using the new parameters.

This paper is organised as follows: In Section 2.1 we describe the N -body simulations used in this paper. In Section 2.2 we discuss two methods for measuring the velocity divergence power spectrum. In Section 3 we present the main results of this paper including the updated parameters for the nonlinear velocity divergence power spectra.

2 MEASURING $P_{\theta\theta}$ AND $P_{\theta\delta}$ FROM N-BODY SIMULATIONS

In section 2.1 we present the details of the N -body simulations carried out and discuss several methods which can be used to measure the velocity divergence power spectrum in Section 2.2. We focus on a Delaunay tessellation approach and show the accuracy which with this method can recover $P_{\theta\theta}$ and $P_{\theta\delta}$.

2.1 Simulation details

We use the N -body simulations carried out by Li *et al.* (2012). These simulations were performed at the Institute of Computational Cosmology using a modified version of the mesh-based N -body code **RAMSES** (Teyssier 2002). Assuming a Λ CDM cosmology, the following cosmological parameters were used in the simulations: $\Omega_m = 0.24$, $\Omega_{DE} = 0.76$, $h = 0.73$ and a spectral tilt of $n_s = 0.961$ (in agreement with e.g. Sánchez *et al.* 2009). The linear theory rms fluctuation in spheres of radius $8 h^{-1} \text{ Mpc}$ is set to be $\sigma_8 = 0.769$.

The simulations use $N = 1024^3$ dark matter particles to represent the matter distribution in a computational box of comoving length $1500h^{-1} \text{ Mpc}$. The initial conditions were generated at $z = 49$ using the MPgrafic code. Note the nonlinear matter power spectrum is measured from the simulations by assigning the particles to a mesh using the cloud in cell (CIC) assignment scheme and performing a fast Fourier transform (FFT) of the density field. To compensate for the mass assignment scheme we perform an approximate deconvolution following Baumgart & Fry (1991).

Throughout this paper the velocity divergence is normalized to $\theta = -\nabla \cdot v / (aHf)$, where v is the peculiar velocity, $f = d\ln\delta/d\ln a$ is the linear growth rate, H is the Hubble parameter and a is the scale factor. Using this normalization θ is dimensionless and the linear continuity equation is given by $\theta = \delta$.

2.2 Measuring the velocity divergence field

Measuring the velocity divergence field accurately from numerical simulations on small scales can be difficult if a mass weighted approach is used as in Scoccimarro (2004); Pueblas & Scoccimarro (2009); Jennings, Baugh & Pascoli (2011a, for example). Some volume weighted measures of the velocity field have also been developed (see e.g. Bernardeau & van de Weygaert 1996; Colombi, Chodorowski & Teyssier 2007) including the Delaunay tessellation field estimator (DTFE) method (Schaap & van de Weygaert 2000; van de Weygaert & Schaap 2009; Cantun & van de Weygaert 2011).

In the mass weighted approach, simply interpolating the velocities to a grid, as suggested by Scoccimarro (2004), gives the momentum field which is then Fourier transformed and divided by the Fourier transform of the density field, which results in a mass weighted velocity field on the grid. One of the main problems with this approach is that the velocity field is artificially set to zero in regions where there are no particles, as the density is zero in these empty cells. Pueblas & Scoccimarro (2009) also found that this method does not accurately recover the input velocity divergence power spectrum on scales $k > 0.2h\text{Mpc}^{-1}$ interpolating the velocities of 640^3 particles to a 200^3 grid. Using simulations of 1024^3 particles in a $1.5h^{-1} \text{ Gpc}$ box, Jennings, Baugh & Pascoli (2011a) found that the maximum grid size that could be used was 350^3 without reaching the limit of empty cells.

The limit on the maximum grid size which can be used in a mass weighted estimate of θ means that the velocity divergence $P(k)$ presented in Jennings, Baugh & Pascoli (2011a) is only accurate on large scales $k < 0.2h\text{Mpc}^{-1}$.

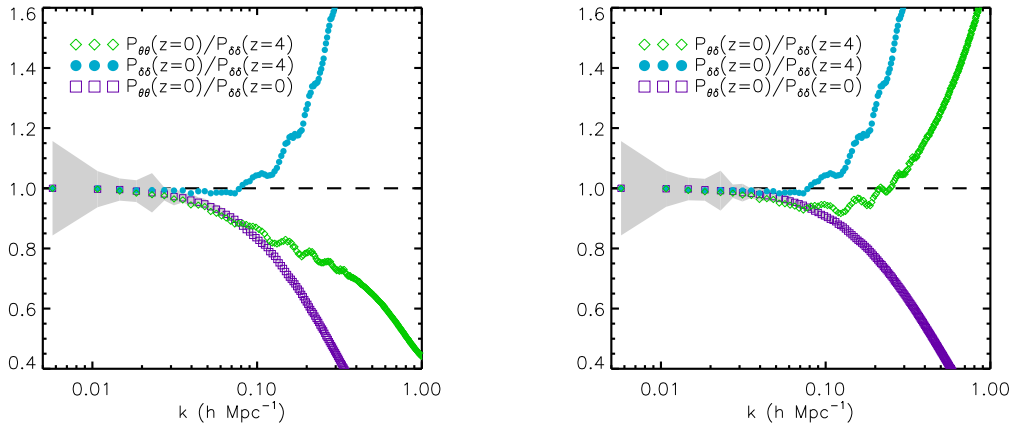


Figure 1. Left panel: Ratios of the $z = 0$ nonlinear matter, $P_{\delta\delta}$, and velocity divergence, $P_{\theta\theta}$, power spectra to the matter power spectrum at $z = 4$, $P_{\delta\delta}(z = 4)$, scaled using the ratio of the square of the linear growth factor at $z = 4$ and $z = 0$ for Λ CDM, measured from six simulations are shown as filled blue circles and empty green diamonds respectively. The ratio of the $P_{\theta\theta}/P_{\delta\delta}$ measured from the simulations at $z = 0$ is shown as empty purple squares. The shaded grey region shows the errors on this ratio measured from the scatter amongst six simulations. Right panel: Similar to the left panel. The ratios $P_{\theta\delta}(z = 0)/P_{\delta\delta}(z = 4)$ and $P_{\theta\delta}(z = 0)/P_{\delta\delta}(z = 0)$ are shown as empty green diamonds and purple squares respectively. Note $\theta = -\nabla \cdot v/(aHf)$.

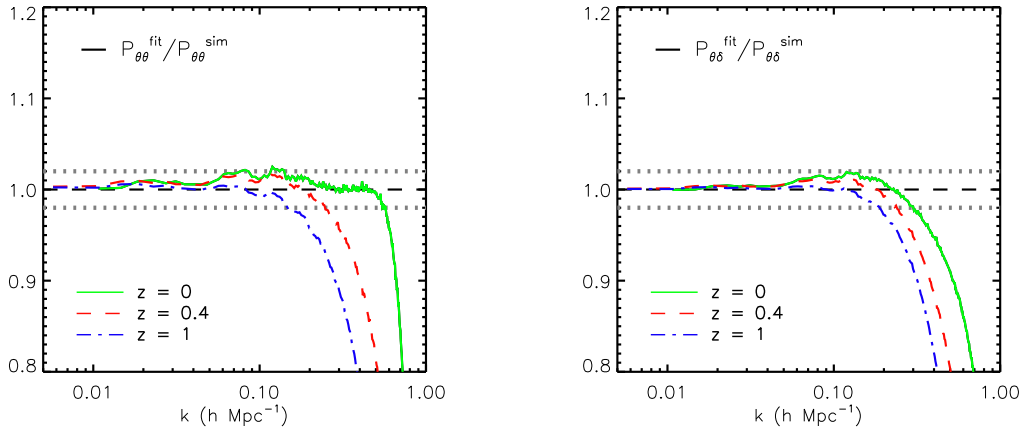


Figure 2. Left panel: The ratio of the fitting formula given in Eq. 1 for the velocity divergence $P(k)$ to the measured power spectrum, $P_{\theta\theta}^{\text{fit}}/P_{\theta\theta}^{\text{sim}}$, at $z = 0$ (green solid), $z = 0.4$ (red dashed) and $z = 1$ (blue dot dashed). Right panel: Similar results as shown in the left panel but for the cross power spectrum $P_{\theta\delta}$. The horizontal dotted grey lines show a region of 2% accuracy in the fit.

Also, it is not clear on what scales the discrepancy between the actual and measured velocity divergence power spectrum, which was found by Pueblas & Scoccimarro (2009) on scales $k < 0.02h\text{Mpc}^{-1}$ (see Figure 13 in that paper), is present in the velocity $P(k)$ measured by Jennings, Baugh & Pascoli (2011a). However, Jennings, Baugh & Pascoli (2011a) find that the measured $P_{\theta\theta}$ and $P_{\theta\delta}$ agree with the linear continuity equation on large scales within the errors from eight simulations (see Figure 3 in that work).

The limitations of using the mass weighted method to measure $P_{\theta\theta}$ implies that the fitting formula presented in Jennings, Baugh & Pascoli (2011a) is only valid over a limited range of scales. The main aim of this paper is to present an improved version of the formula for $P_{\theta\theta}$ and $P_{\theta\delta}$ given by Jennings, Baugh & Pascoli (2011a) by fitting to power spectra which have been measured using the DTFE method. This code constructs the Delaunay tessellation from a discrete set of points and interpolates the field values onto a user defined

grid. For the $L_{\text{box}} = 1500h^{-1}$ Mpc simulation we generate the velocity auto, $P_{\theta\theta}$, and cross power spectrum, $P_{\theta\delta}$, on a 1024^3 grid. The velocity divergence field is interpolated onto the grid by randomly sampling the field values at a given number of sample points within the Delaunay cells and then taking the average of those values. The resolution of the mesh used in this study means that mass assignment effects are negligible on the scales of interest here.

In Fig. 1 we show the ratios of $P_{\delta\delta}(z = 0)/P_{\delta\delta}(z = 4)$ and $P_{\theta\theta}(z = 0)/P_{\delta\delta}(z = 4)$ which represent the average ratio measured from six simulations as filled blue circles and empty green diamonds respectively. Here $P_{\delta\delta}(z = 4)$ has been scaled using the ratio of the square of the linear growth factor at $z = 4$ and $z = 0$ for Λ CDM. Plotting the ratio in this way removes the sampling variance (Baugh & Efstathiou 1994) and shows the agreement of the measured velocity and matter $P(k)$ with the predictions of the linear continuity equation on large scales, $k < 0.02h\text{Mpc}^{-1}$ for $P_{\theta\theta}(z = 0)/P_{\delta\delta}(z = 4)$ and $k <$

$0.09h\text{Mpc}^{-1}$ for $P_{\delta\delta}(z=0)/P_{\delta\delta}(z=4)$. This result agrees with similarly measured ratios from simulations for the matter (see e.g. Angulo *et al.* 2008) and the velocity divergence power spectrum (Jennings, Baugh & Pascoli 2011a; Jennings *et al.* 2012; Li *et al.* 2012).

The ratio of $P_{\theta\theta}(z=0)/P_{\delta\delta}(z=0)$ is plotted in the left panel of Fig. 1 as empty purple circles. The grey shaded region shows the errors measured from the scatter amongst six simulations. In the right panel of Fig. 1 we plot similar ratios as in the left panel but for the velocity divergence cross power spectrum $P_{\theta\delta}$. Note the increase in the ratio of $P_{\theta\delta}(z=0)/P_{\delta\delta}(z=4)$ on scales $k > 0.1h\text{Mpc}^{-1}$ which is due to the nonlinear growth in the matter field on these scales. Overall the results plotted in Fig. 1 show that the velocity divergence field computed using the DTFE method agrees with the predictions of linear theory on extremely large scales and allows us to accurately measure the power spectra on nonlinear scales provided we use a large enough grid to interpolate the velocities. We have checked that the effect on the measured velocity power spectra using smaller grid sizes such as 256^3 and 512^3 and find excellent agreement between the $P(k)$ measured using a 512^3 and 1024^3 grid on scales $k < 1h\text{Mpc}^{-1}$; the $P(k)$ measured using a 256^3 grid shows deviations at $k \sim 0.7h\text{Mpc}^{-1}$.

3 RESULTS

Using the measured nonlinear velocity divergence auto and cross power spectrum on a 1024^3 grid presented in Section 2.2 and the nonlinear matter power spectrum measured from the simulations at $z=0$, we can fit for new parameters in the following model (Jennings, Baugh & Pascoli 2011a):

$$P_{xy}(k) = g(P_{\delta\delta}(k)) = \frac{\alpha_0 \sqrt{P_{\delta\delta}(k)} + \alpha_1 P_{\delta\delta}^2(k)}{\alpha_2 + \alpha_3 P_{\delta\delta}(k)}, \quad (1)$$

where $P_{\delta\delta}$ is the nonlinear matter power spectrum. We obtain the following:

- $P_{xy} = P_{\delta\theta}$:
 $\alpha_0 = -12483.8$, $\alpha_1 = 2.554$, $\alpha_2 = 1381.29$, $\alpha_3 = 2.540$;
- $P_{xy} = P_{\theta\theta}$:
 $\alpha_0 = -12480.5$, $\alpha_1 = 1.824$, $\alpha_2 = 2165.87$, $\alpha_3 = 1.796$;

all points were weighted equally in the fit. Note the parameters α_{0-3} are not dimensionless and in this work their units differ from those quoted in Jennings, Baugh & Pascoli (2011a) as the best fit parameters presented here generate velocity divergence power spectra which are normalized as $\theta = \nabla \cdot v / (aHf)$. Note the growth rate at $z=0$ for the ΛCDM cosmology considered in this work is $f = 0.452$. We do not give the units of α_{0-3} here but these can be easily found given this normalisation for theta. The power spectra used for this fit are the average $P_{\theta\theta}$, $P_{\theta\delta}$ and $P_{\delta\delta}$ measured from six ΛCDM simulations.

In Fig. 2 we plot the ratio of $P_{\theta\theta}^{\text{fit}}/P_{\theta\theta}^{\text{sim}}$ and $P_{\theta\delta}^{\text{fit}}/P_{\theta\delta}^{\text{sim}}$ in the left and right panels respectively at $z=0$ as a green solid line. Here the prefix ‘fit’ denotes the velocity divergence power spectrum found using the new best fit parameters and the nonlinear matter $P(k)$ measured from the simulations in Eq. 1. The prefix ‘sim’ denotes the nonlinear velocity divergence measured directly from the simulations using the DTFE. The formula given in Eq. 1 is accurate to 2%

(shown as the region enclosed by the dotted gray lines in Fig. 2) on scales $k < 0.65h\text{Mpc}^{-1}$ for $P_{\theta\theta}$ and $k < 0.35h\text{Mpc}^{-1}$ for $P_{\theta\delta}$ at $z=0$. Note the lower limit for the domain of the function given in Eq. 1 is $k \geq 0.006h\text{Mpc}^{-1}$. Although the values for α_{0-3} were obtained by fitting to a ΛCDM simulation with a particular set of cosmological parameters, Jennings, Baugh & Pascoli (2011a) found that this relation between the density and velocity power spectra is quite insensitive to both the cosmological model, for smooth dark energy models such as quintessence, and the choice of cosmological parameters. These results agree with previous studies such as Bernardeau (1992) and Bouchet *et al.* (1995).

The redshift dependence of $P_{\theta\theta}$ and $P_{\theta\delta}$ can be described using the following formula (Jennings, Baugh & Pascoli 2011a):

$$P_{xy}(k, z') = \frac{g(P_{\delta\delta}(k, z=0)) - P_{\delta\delta}(k, z=0)}{c^2(z=0, z')} + P_{\delta\delta}(k, z'), \quad (2)$$

where $g(P_{\delta\delta})$ is the function in Eq. 1 and P_{xy} is either the nonlinear cross or auto power spectrum, $P_{\delta\theta}$ or $P_{\delta\delta}$ and the function c is given by

$$c(z, z') = \frac{D(z) + D^2(z) + D^3(z)}{D(z') + D^2(z') + D^3(z')}, \quad (3)$$

and $D(z)$ is the linear growth factor.

In Fig. 2 we show the ratio of $P_{\theta\theta}^{\text{fit}}$ from Eqns. 1 & 2 to the measured velocity divergence power spectrum $P_{\theta\theta}^{\text{sim}}$ at $z=0.4$ (red dashed) and $z=1$ (blue dot dashed). All of the results shown in this plot represent the average over six ΛCDM simulations. It is clear from Fig. 2 that the model given in Eq. 2 is accurate to better than 2% on scales $k < 0.2h\text{Mpc}^{-1}$ and to 20% for $0.2 < k(h\text{Mpc}^{-1}) < 0.4$ at $z=1$.

4 SUMMARY

Measuring the growth of structure using the anisotropic clustering signal in redshift space is an important tool for discriminating between dynamical dark energy or modified gravity and the standard ΛCDM cosmological model. The velocity divergence power spectrum has been shown to be an important ingredient in modelling redshift space distortions (Scoccimarro 2004; Jennings, Baugh & Pascoli 2011a) and shows larger deviations between modified gravity cosmologies and ΛCDM than the differences found in the nonlinear matter power spectrum using numerical simulations (see e.g. Schmidt *et al.* 2009b; Jennings *et al.* 2012; Li *et al.* 2012).

In this paper we present an improved model for the $z=0$ velocity divergence auto and cross power spectrum which was originally suggested by Jennings, Baugh & Pascoli (2011a). By measuring the velocity fields using a Delaunay tessellation we obtain an accurate prediction of the velocity $P(k)$ to $k \sim 1h\text{Mpc}^{-1}$ which we use to update the parameters for the fitting formula given in Eq. 1. We find that this model is accurate to 2% on scales $k < 0.65h\text{Mpc}^{-1}$ for $P_{\theta\theta}$ and $k < 0.35h\text{Mpc}^{-1}$ for $P_{\theta\delta}$ at $z=0$. We find that the formula for the redshift dependence of the velocity power spectra given in Jennings, Baugh & Pascoli (2011a) recovers the $z > 0$ $P(k)$ to markedly greater accuracy with the new parameters. This model for the redshift evolution of $P_{\theta\theta}$

and $P_{\theta\theta}$, given in Eq. 2, is accurate to less than 2% on scales $k < 0.2h\text{Mpc}^{-1}$ and to 20% for $0.2 < k(h\text{Mpc}^{-1}) < 0.4$ at $z = 1$.

The improved model presented here accurately describes the nonlinear density velocity relation and allows the velocity divergence power spectra to be easily and accurately predicted over a range of scales $k < 0.9h\text{Mpc}^{-1}$ and redshifts $z \leq 1$. This model will be useful for analysing measurements of peculiar velocities at high and low redshifts (e.g. Hudson & Turnbull 2012) and for studying the redshift space clustering signal in galaxy redshift surveys. This improved model has already been implemented in the analysis of the 6dF galaxy redshift survey (Beutler *et al.* 2012).

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